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AN INVESTIGATION INTO THE REAL GAS EFFECTS OF CRYOGENIC NITROGE--ETC(U)
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AN INVESTIGATION INTO THE REAL GAS EFFECTS OF CRYOGENIC NITROGEN IN INVISCID HOMENTROPIC FLOW

by

C. M. Albone

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(10) C. M. Albone

SUMMARY

As a contribution to the investigation of the suitability of using cryogenic nitrogen as the test gas in a high Reynolds number transonic wind-tunnel, a study is made here of the real gas effects of nitrogen at low temperatures. The study, which is limited to inviscid, homentropic flow of a non-conducting gas, takes the form of an independent confirmation of results by Kilgore *et al*[1]. A recent paper by Wagner and Schmidt[2] on this subject employs a different equation of state from that used here and their investigations cover more than just homentropic flow. The new contribution in this Memorandum is that the use of a simplified equation of state enables an expression for enthalpy (and hence the terms in Bernoulli's equation) to be derived by analytic integration.

A shortened version of this Memorandum was presented at the First International Symposium on Cryogenic Wind-Tunnels at Southampton University, 3-5 April 1979.

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1 INTRODUCTION

A proposal for a new high Reynolds number wind-tunnel involves the use of very cold nitrogen[1] (called cryogenic nitrogen) as a test gas. The much reduced viscosity and increased density at low temperatures gives such a facility a high Reynolds number capability without the need for extreme pressurisation or the use of large scale models. We wish to determine to what extent the pressure distribution over a configuration tested in cold nitrogen differs from that for the same configuration in warm air. Some differences may be anticipated, since it is known[3] that the equation of state for a gas at low temperatures departs from that of a perfect gas.

As a contribution to the investigation of these real gas effects, a study is made of the equations governing the flow of cold nitrogen in a wind-tunnel to see how they differ from those governing the flow of warm air. In order to simplify this study, certain assumptions are made regarding the nature of the flow. The author has been unable to find any evidence which shows that vibrational or rotational relaxation effects are significant, and so accordingly these are neglected, as too are the effects of heat conduction. Since we wish to consider high Reynolds numbers, we will assume the flow to be inviscid and accept that any conclusions we may draw apply to flows without large scale separation, and then only to the region outside the boundary layer. Our study will further be restricted by considering the flow to be steady and irrotational and hence (with a uniform flow far upstream) homentropic*. This restriction prohibits a consideration of flows with shock waves, although, if these are weak, any conclusions may carry over in an approximate way. The existence of a velocity potential, however, reduces the problem to that of solving a second-order partial differential equation (called the potential equation) for the velocity potential ϕ, coupled with one or more auxiliary equations for the speed of sound, a. The form of the potential equation is independent of real gas effects, since it is derived from the laws of conservation of mass and momentum. The auxiliary equations are however dependent upon real gas effects. In general, they consist of Bernoulli's equation and the equation of state together with a constraint provided by the condition of homentropic flow.

* The word homentropic is used in this Memorandum to describe a flow in which there are no variations of entropy. Many readers would prefer the word isentropic, which means that entropy is constant along a streamline, but it does not imply the existence of a velocity potential.

We confine our investigations to a study of these auxiliary equations, and the starting point for this study is an equation of state for nitrogen[3] under a wide range of conditions. In section 2, this equation of state is simplified for the restricted temperature and pressure range of a cryogenic wind-tunnel. Expressions for specific heats and the speed of sound are derived in sections 3, 4 and 5. Numerical values for some of these parameters under cryogenic conditions are given in section 6, together with an approximate homentropic expansion law derived from these. The exact homentropic flow condition is derived in section 7, and, in section 8, an exact form of Bernoulli's equation is given. The near-ideal behaviour of cryogenic nitrogen is illustrated in a one-dimensional expansion through a nozzle described in section 9. Some concluding remarks appear in section 10.

2 A SIMPLIFIED FORM OF JACOBSEN'S[3] EQUATION OF STATE FOR NITROGEN

Jacobsen's[3] 32-parameter equation of state for nitrogen is valid for pressures in the range 1 atm to 10000 atm and temperatures in the range 65 K to 2000 K. An approximation to Jacobsen's equation has been derived (for the temperature range 100 K → 300 K and for the pressure range 1 atm → 5 atm), which takes the form of the ideal gas equation of state plus the first term of an expansion in powers of ρ

$$p = \rho RT + \rho^2 f(T) , \tag{1}$$

where $f(T) = N_1 T + N_2 T^{\frac{1}{2}} + N_3 + N_4/T + N_5/T^2$, p is the pressure in atmospheres, ρ is the density in moles/litre, T is the temperature in kelvins and R is the gas constant, taken as $R = 0.082$. The coefficients N_1-N_5 take the following values:

$$N_1 = 0.00136 \; ; \quad N_2 = 0.10703 \; ; \quad N_3 = -2.439$$

$$N_4 = 34.10 \; ; \quad N_5 = -4223.74 \; .$$

At the extreme conditions of $T = 100$ K and $p = 5$ atm, equation (1) gives $(p - \rho RT)/p$ to within 2% of Jacobsen's values (that is to say within 2% of the *deviation* from a perfect gas). The advantages to be gained from having an equation of state in the simplified form (1) will be apparent in later sections of this Memorandum.

3 EVALUATION OF SPECIFIC HEAT AT CONSTANT VOLUME (C_v)

Equation (38) on page 45 of Howarth[4] implies that

$$\left(\frac{\partial C_v}{\partial \rho}\right)_T = -\frac{T}{\rho^2}\left(\frac{\partial^2 p}{\partial T^2}\right)_\rho \quad . \tag{2}$$

Using equation (1), we have

$$\left(\frac{\partial p}{\partial T}\right)_\rho = \rho R + \rho^2 f' \ , \tag{3}$$

where f' denotes df/dT, and so

$$\left(\frac{\partial^2 p}{\partial T^2}\right)_\rho = \rho^2 f'' \quad .$$

Thus from equation (2) we have

$$\left(\frac{\partial C_v}{\partial \rho}\right)_T = -\, Tf''$$

and so on integration

$$C_v = -\, Tf''\rho + g(T) \ ,$$

where $g(T)$ is a function of T which is tabulated by Jacobsen[3]. In the range $T = 100\ K \rightarrow 300\ K$, $g(T)$ does not depart from the (ideal diatomic) value of 2.5R by more than 0.03%. We accept this approximation and adopt the following expression for C_v:

$$C_v = 2.5R - T\rho f'' \quad . \tag{4}$$

4 EVALUATION OF SPECIFIC HEAT AT CONSTANT PRESSURE (C_p) AND γ

Equation (32) on page 45 of Howarth[4] implies that

$$C_p - C_v = -\frac{T}{\rho^2}\left(\frac{\partial p}{\partial T}\right)_\rho \left(\frac{\partial \rho}{\partial T}\right)_p \quad . \tag{5}$$

Since pressure is a function of ρ and T, we may in general write

$$dp = \left(\frac{\partial p}{\partial \rho}\right)_T d\rho + \left(\frac{\partial p}{\partial T}\right)_\rho dT \quad . \tag{6}$$

Hence

$$\left(\frac{\partial \rho}{\partial T}\right)_p = -\left(\frac{\partial p}{\partial T}\right)_\rho \bigg/ \left(\frac{\partial p}{\partial \rho}\right)_T \quad .$$

On substituting this expression into equation (5), we obtain

$$C_p - C_v = \frac{T}{\rho^2}\left(\frac{\partial p}{\partial T}\right)_\rho^2 \bigg/ \left(\frac{\partial p}{\partial \rho}\right)_T \quad . \tag{7}$$

From equation (1), we have

$$\left(\frac{\partial p}{\partial \rho}\right)_T = RT + 2\rho f \ . \tag{8}$$

Substituting from equations (3) and (8) into (7), we have

$$C_p - C_v = \frac{T}{\rho^2}\,\frac{(\rho R + \rho^2 f')^2}{(RT + 2\rho f)} \ ,$$

which using equation (4) becomes

$$C_p = 2.5R - T\rho f'' + \frac{T(R + \rho f')^2}{(RT + 2\rho f)} \quad . \tag{9}$$

The ratio of specific heats, γ, may thus be evaluated as

$$\gamma = \frac{C_p}{C_v} \tag{10}$$

using equations (4) and (9).

5 EVALUATION OF a^2

Equation (6) implies that

$$\left(\frac{\partial p}{\partial \rho}\right)_S = \left(\frac{\partial p}{\partial \rho}\right)_T + \left(\frac{\partial p}{\partial T}\right)_\rho \left(\frac{\partial T}{\partial \rho}\right)_S ,$$

where S is the entropy, and so by definition, the speed of sound, a, is given by

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_T + \left(\frac{\partial p}{\partial T}\right)_\rho \left(\frac{\partial T}{\partial \rho}\right)_S . \tag{11}$$

Equation (35) on page 45 of Howarth[4] implies that

$$dS = \frac{C_v dT}{T} - \frac{1}{\rho^2}\left(\frac{\partial p}{\partial T}\right)_\rho d\rho .$$

Hence

$$\left(\frac{\partial T}{\partial \rho}\right)_S = \frac{T}{\rho^2 C_v}\left(\frac{\partial p}{\partial T}\right)_\rho . \tag{12}$$

On substituting for $(\partial T/\partial \rho)_S$ from equation (12) and for $(\partial p/\partial T)_\rho$ from equation (7) into equation (11), we obtain

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_T + \left(\frac{C_p - C_v}{C_v}\right)\left(\frac{\partial p}{\partial \rho}\right)_T ,$$

which simplifies to

$$a^2 = \gamma \left(\frac{\partial p}{\partial \rho}\right)_T , \tag{13}$$

and using equation (8) this becomes

$$a^2 = \gamma(RT + 2\rho f) . \tag{14}$$

6 A SUGGESTION OF A POSSIBLE SIMPLE SOLUTION FOR COLD NITROGEN

We consider a set of values of T given by

$$T = 100\ (5)\ 120\ (10)\ 160\ (20)\ 300\ \text{K}$$

and a set of values of p for each value of T given by

$$p = 1\ (1)\ 5\ \text{atm} .$$

For each value of p and T, the values of ρ, γ and a^2 are evaluated from equations (1), (10) and (14) respectively, together with the compressibility factor Z defined by

$$p = \rho RTZ$$

so that

$$Z = 1 + \frac{\rho f}{RT} . \qquad (15)$$

The variation of Z with T is shown in Fig 1 for values of $p = 1$ atm and 5 atm. In Fig 2, the variation of γ with T is shown for the same two values of p. Its departure from the ideal value of 1.4 is considerable at the lowest temperatures. Shown in Fig 3 is the variation with T of α, where α is defined as

$$a^2 = \frac{\alpha p}{\rho} . \qquad (16)$$

For cryogenic nitrogen, we see that even though γ departs significantly from 1.4 at $p = 5$ atm as T is reduced towards 100 K, the quantity α remains closer to a value of 1.4. Consider the consequence of assuming that α is constant. We are then able to integrate equation (16), using the definition of a^2, to obtain the homentropic relation as

$$\frac{p}{\rho^\alpha} = \text{const.} \qquad (17)$$

Hence, if α is constant, it is the homentropic expansion coefficient. Bernoulli's equation for a steady inviscid non-conducting flow is

$$h + \tfrac{1}{2}q^2 = \int \frac{dp}{\rho} + \tfrac{1}{2}q^2 = \text{const} , \qquad (18)$$

where h is the enthalpy.
This would become, using equations (17) and (16),

$$\frac{a^2}{\alpha - 1} + \tfrac{1}{2}q^2 = \text{const} , \qquad (19)$$

where q is the local speed of the flow.
Hence under the assumption that α is a constant and that it takes a value of 1.4, Bernoulli's equation takes the same form as that for an ideal diatomic gas. Our assumption has used numerical results to suggest a relation (16) which enables an approximate homentropic expansion law (17) to be postulated resulting in a simple form for Bernoulli's equation (19). The argument for the existence of a simple form of Bernoulli's equation, like (19) would be more convincing if numerical approximations were not employed until (if at all) a much later stage. To this end, we next derive the exact homentropic expansion relation for cryogenic nitrogen.

7 THE HOMENTROPIC FLOW RELATION FOR CRYOGENIC NITROGEN

For homentropic flow, equation (12) implies that

$$\frac{C_v dT}{T} = \frac{1}{\rho^2}\left(\frac{\partial p}{\partial T}\right)_\rho d\rho .$$

Using equations (3) and (4) for $(\partial p/\partial T)\rho$ and C_v respectively, we have

$$\left\{\frac{2.5R - \rho T f''}{T}\right\} dT = \left\{\frac{\rho R + \rho^2 f'}{\rho^2}\right\} d\rho ,$$

which becomes on integrating,

$$2.5R \ln T - \int \rho f'' dT = R \ln \rho + \int f' d\rho .$$

Integrating the last term on the right hand side by parts and rearranging gives

$$R \ln\left(\frac{T}{\rho^{0.4}}\right) = 0.4\rho f' + \text{const.} \qquad (20)$$

Numerical results for a typical homentropic expansion in a cryogenic wind-tunnel show that equation (20), when combined with the equation of state (1), is very closely approximated by equation (17) with $\alpha = 1.4$. This evidence is encouraging, but we would like to take the analysis one step further.

8 AN EXACT FORM OF BERNOULLI'S EQUATION FOR CRYOGENIC NITROGEN

We wish to evaluate the enthalpy, $h\left(=\int(dp/\rho)\right)$, in Bernoulli's equation (18), for the equation of state (1)

$$p = \rho RT + \rho^2 f(T)$$

subject to the homentropic condition (20)

$$R \ln\left(\frac{T}{\rho^{0.4}}\right) = 0.4\rho f' + K_1 ,$$

where K_1 is a constant. Integrating by parts, we have

$$h = \int\frac{dp}{\rho} = \left[\frac{p}{\rho}\right] - \int p d\left(\frac{1}{\rho}\right) = \left[\frac{p}{\rho}\right] + \int\frac{p}{\rho^2}\,d\rho .$$

Substituting for p from equation (1), we obtain

$$h = \left[\frac{p}{\rho}\right] + \int\frac{RT}{\rho}\,d\rho + \int f d\rho ,$$

which rearranges to become

$$h = \left[\frac{p}{\rho}\right] + \int RTd(\ln\rho) + \int f d\rho .$$

Integrating each term by parts again gives

$$h = \left[\frac{p}{\rho} + RT\ln\rho + \rho f\right] - \int(R\ln\rho + \rho f')dT .$$

From equation (20) we have

$$\int(R\ln\rho + \rho f')dT = \int 2.5R\ln T dT$$

and so we can eliminate $R \ln \rho + \rho f'$ and obtain

$$h = \left[\frac{p}{\rho} + RT \ln \rho + \rho f\right] - 2.5R \int \ln T dT$$

$$= \frac{p}{\rho} + RT \ln \rho + \rho f - 2.5R(T \ln T - T) + \text{const.}$$

Using again equation (20) and the equation of state (1), this can be reduced to

$$h = 3.5RT + 2\rho f - \rho T f' + \text{const} . \tag{21}$$

This simple analytic form for enthalpy for cryogenic nitrogen does not appear to have been presented before. We may now write Bernoulli's equation (18) for the homentropic flow of a gas governed by the equation of state (1) as

$$3.5RT + 2\rho f - \rho T f' + \tfrac{1}{2}q^2 = \text{const} . \tag{22}$$

From equations (4), (9), (10) and (14) for C_ρ, C_v, γ and a^2 respectively, we may write a^2 as

$$a^2 = RT + 2\rho f + \frac{T(R + \rho f')^2}{2.5R - \rho T f''} . \tag{23}$$

Hence we may introduce a^2 into Bernoulli's equation (with no approximations other than those implied by the equation of state (1)) to obtain

$$\frac{a^2}{\beta - 1} + \tfrac{1}{2}q^2 = \text{const} , \tag{24}$$

where $\beta = 1 + \dfrac{RT + 2\rho f + \dfrac{T(R + \rho f')^2}{2.5R - \rho T f''}}{3.5RT + 2\rho f - \rho T f'}$.

Values of β have been obtained for the same range of T and p as had been used for the evaluation of α in section 6. At $p = 1$ atm, β increases monotonically with T from 1.398 at $T = 100$ K to 1.401 at $T = 300$ K. At $p = 5$ atm, β increases again monotonically from 1.387 at $T = 100$ K to 1.403 at $T = 300$ K. Values of β for the latter case are plotted in Fig 3, but for $p = 1$ atm the variation is too small to be plotted, and is omitted. It is noted

that values of β vary less than those of α in the cryogenic range, and nowhere in that range do they exceed ±1% of the ideal diatomic value of 1.4.

9 ONE-DIMENSIONAL FLOW IN A CONVERGENT-DIVERGENT NOZZLE

To illustrate the near-ideal behaviour of cryogenic nitrogen in homentropic flow, we consider the one-dimensional expansion of nitrogen from typical wind-tunnel stagnation conditions to a supersonic flow near to the saturation boundary. If A is the cross-sectional area of the nozzle, then by conservation of mass we have

$$\rho q A = \text{const.} = K_3 . \tag{25}$$

Bernoulli's equation (22) and the homentropic condition (20) complete the set of three equations for ρ, q and T corresponding to a given area distribution. The elimination of q between equations (22) and (25) results in

$$3.5RT + 2\rho f - \rho T f' + \frac{1}{2}\left(\frac{K_1}{\rho A}\right)^2 = K_2 ,$$

which together with equation (20),

$$R \ln\left(\frac{T}{\rho^{0.4}}\right) = 0.4\rho f' + K_1 ,$$

forms a pair of coupled implicit algebraic equations for ρ and T as functions of A, where K_1, K_2 and K_3 are constants. These constants can be evaluated from given reference conditions and then the equations solved by a double-iterative procedure. SI units are chosen for this exercise. The gas constant, R, becomes 296.813 $m^2\ s^{-2}$, and the five constants in the equation of state (1) become:

$$\begin{aligned} N_1 &= 0.17572 \\ N_2 &= 13.829 \\ N_3 &= -\ 315.14 \\ N_4 &= 4406.06 \\ N_5 &= -\ 545749.45. \end{aligned}$$

The proposed stagnation conditions[5] for the European cryogenic facility were considered, namely

$$p_0 = 4.4\ \text{atm} \quad \text{and} \quad T_0 = 120\ \text{K} .$$

For the calculation, reference conditions used to determine K_1, K_2 and K_3 were not taken as those at stagnation because of numerical problems. Reference conditions for p, T, q and A were obtained by an iterative process which converged only when the target stagnation conditions were met to an acceptable accuracy. The actual values of p_0 and T_0 used in the calculation are

$$p_0 = 4.3954 \text{ atm} \quad \text{and} \quad T_0 = 119.96 \text{ K} .$$

The expansion was carried out through the locally sonic conditions to a Mach number, M , of 1.65 just beyond the saturation boundary[1]. Table 1 shows values of q, p, ρ, T and M for given values of A , together with p/p_0, ρ/ρ_0 and T/T_0 corresponding to A^*/A where A^* is the value of A at the sonic condition. Also shown in Table 1 are the values of p/p_0, ρ/ρ_0 and T/T_0 corresponding to the same A^*/A for air as an ideal gas. In Fig 4, values of temperature and pressure extracted from the table are plotted against Mach number. The departure (D) from an ideal diatomic gas, defined as

$$D = \left\{ \left(\frac{p}{p_0}\right)_{\text{nitrogen}} - \left(\frac{p}{p_0}\right)_{\substack{\text{ideal gas at the}\\ \text{same area ratio}}} \right\} \Bigg/ \left(\frac{p}{p_0}\right)_{\text{nitrogen}}$$

is plotted in Fig 5 against Mach number for pressure and temperature, assuming a similar expression for the departure for temperature. The departure for density is less than that for pressure or temperature over the whole range.

10 CONCLUDING REMARKS

In this Memorandum, it has been shown that the thermal and calorific imperfections which give rise to a significant variation in γ are such that, for the homentropic flow of a non-conducting gas, Bernoulli's equation takes a form which may be closely approximated by that for an ideal diatomic gas. This form of Bernoulli's equation has been derived using a simplified form of the equation of state, which enables an expression for enthalpy to be derived by analytic integration. The behaviour of nitrogen under typical cryogenic wind-tunnel conditions is illustrated by a calculation of the one-dimensional flow through a convergent-divergent nozzle. The departure from ideal diatomic gas behaviour is less than 0.6% over the whole range of interest.

Table 1

ONE-DIMENSIONAL FLOW IN A CONVERGENT-DIVERGENT NOZZLE

Cryogenic nitrogen expanded from stagnation conditions ($T \simeq 120$ K, $p \simeq 4.4$ atm) to a local Mach number of 1.65											Air as an ideal gas at the same area ratio		
Area (m^2) A	Speed ($m\ s^{-1}$) q	Pressure (atm) $\times 10^5 .p$	Pressure ($N\ m^{-2}$) p	Density ($kg\ m^{-3}$) ρ	Temperature (K) T	Mach number M	A^*/A	p/p_0	ρ/ρ_0	T/T_0	p/p_0	ρ/ρ_0	T/T_0
∞	0.0	4.3954	4.4526	13.210	119.96	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0
10.0	11.66	4.3866	4.4436	13.191	119.89	0.0537	0.0927	0.9980	0.9986	0.9994	0.9980	0.9986	0.9994
3.7	31.80	4.3298	4.3861	13.068	119.44	0.1469	0.2506	0.9851	0.9893	0.9957	0.9850	0.9893	0.9958
2.0	60.54	4.1611	4.2152	12.702	118.06	0.2812	0.4637	0.9467	0.9615	0.9842	0.9465	0.9615	0.9844
1.35	95.11	3.8334	3.8832	11.977	115.26	0.4471	0.6869	0.8721	0.9067	0.9608	0.8718	0.9066	0.9615
1.1	126.03	3.4441	3.4889	11.092	111.71	0.6017	0.8430	0.7836	0.8397	0.9312	0.7830	0.8397	0.9324
0.97	160.17	2.9385	2.9767	9.898	106.66	0.7823	0.9560	0.6685	0.7493	0.8891	0.6676	0.7494	0.8909
0.94	177.08	2.6691	2.7038	9.238	103.72	0.8769	0.9865	0.6072	0.6993	0.8646	0.6057	0.6993	0.8667
0.93	188.37	2.4858	2.5181	8.778	101.60	0.9423	0.9971	0.5655	0.6645	0.8469	0.5640	0.6647	0.8493
0.928	193.19	2.4069	2.4382	8.578	100.65	0.9709	0.9993	0.5476	0.6494	0.8390	0.5460	0.6493	0.8410
0.9275	195.60	2.3674	2.3982	8.476	100.17	0.9853	0.9998	0.5386	0.6416	0.8350	0.5370	0.6414	0.8371
0.9274	196.51	2.3527	2.3833	8.438	99.98	0.9908	0.99993	0.5353	0.6422	0.8334	0.5337	0.6420	0.8355
0.92736	197.11	2.3427	2.3732	8.413	99.86	0.9944	0.99997	0.5330	0.6369	0.8324	0.5315	0.6368	0.8346
0.92734	197.61	2.3346	2.3650	8.392	99.76	0.9974	0.99999	0.5311	0.6353	0.8316	0.5296	0.6352	0.8338
0.9273365	198.02	2.3287	2.3590	8.375	99.69	1.0	1.0	0.5298	0.6340	0.8310	0.5283	0.6339	0.8333
0.92734	198.41	2.3214	2.3516	8.3577	99.60	1.0023	0.99999	0.5281	0.6327	0.8303	0.5266	0.6326	0.8280
0.92736	199.03	2.3115	2.3415	8.3318	99.47	1.0060	0.99997	0.5259	0.6307	0.8292	0.5244	0.6306	0.8269
0.9274	199.54	2.3031	2.3330	8.3101	99.37	1.0091	0.99993	0.5240	0.6291	0.8284	0.5225	0.6290	0.8260
0.9275	200.44	2.2883	2.3180	8.2716	99.18	1.0146	0.9998	0.5206	0.6262	0.8268	0.5191	0.6261	0.8244
0.928	202.88	2.2483	2.2775	8.1678	98.67	1.0296	0.9993	0.5115	0.6183	0.8225	0.5100	0.6182	0.8200
0.93	207.74	2.1689	2.1971	7.9596	97.65	1.0596	0.9971	0.4934	0.6025	0.8140	0.4919	0.6024	0.8165
0.94	219.15	1.9836	2.0094	7.4651	95.15	1.1321	0.9865	0.4513	0.5651	0.7932	0.4497	0.5650	0.7958
0.95	226.20	1.8703	1.8946	7.1562	93.54	1.1783	0.9761	0.4255	0.5417	0.7798	0.4239	0.5417	0.7825
0.97	236.44	1.7082	1.7304	6.7050	91.12	1.2476	0.9560	0.3886	0.5076	0.7596	0.3872	0.5077	0.7625
1.0	247.68	1.5348	1.5548	6.2087	88.34	1.3268	0.9273	0.3492	0.4700	0.7364	0.3776	0.4702	0.7394
1.05	261.52	1.3296	1.3469	5.6002	84.74	1.4296	0.8832	0.3025	0.4239	0.7064	0.3010	0.4240	0.7095
1.12	275.88	1.1282	1.1429	4.9769	80.82	1.5434	0.8280	0.2567	0.3768	0.6737	0.2551	0.3770	0.6769
1.2	288.46	0.9634	0.9759	4.4425	77.22	1.6500	0.7728	0.2192	0.3363	0.6437	0.2177	0.3365	0.6469

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No.	Author	Title, etc
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5		Definition of the minimum operating temperature for preliminary design of ETW. Technical Group European Transonic Wind Tunnel, April 1978

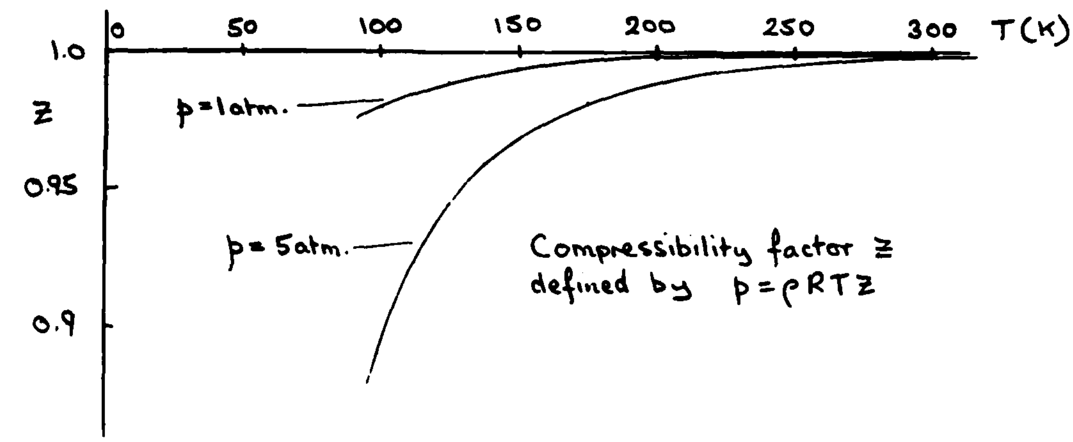

Fig 1 Variation of compressibility factor (Z) with T

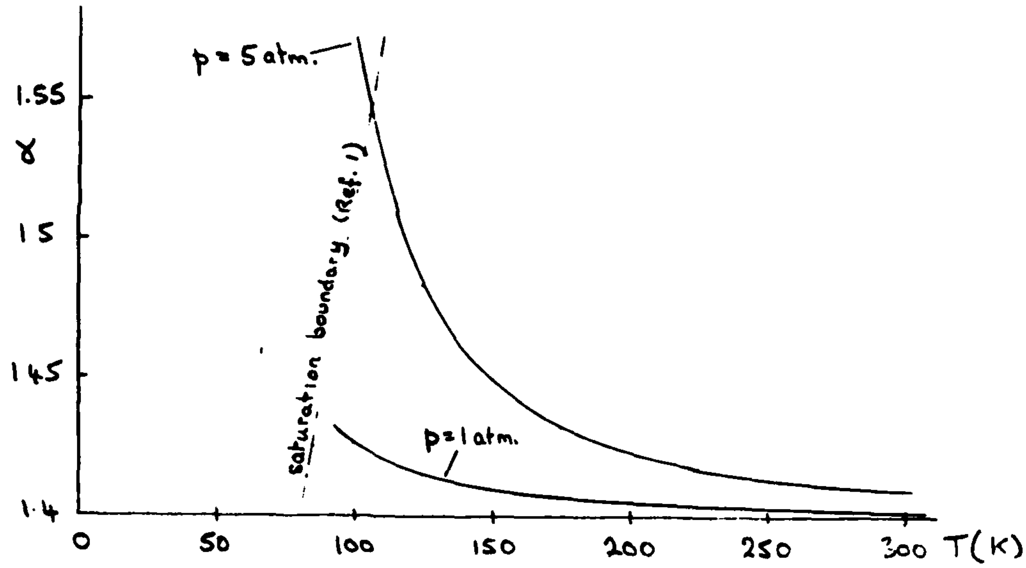

Fig 2 Variation of γ with T

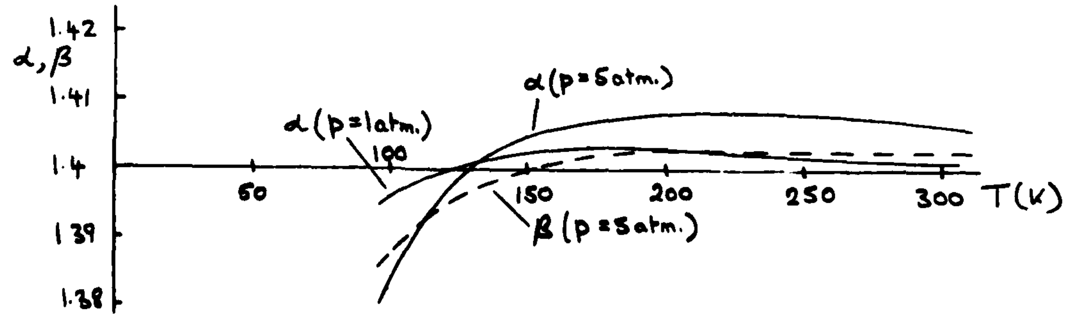

Fig 3 Variation of α and β with T

Figs 4&5

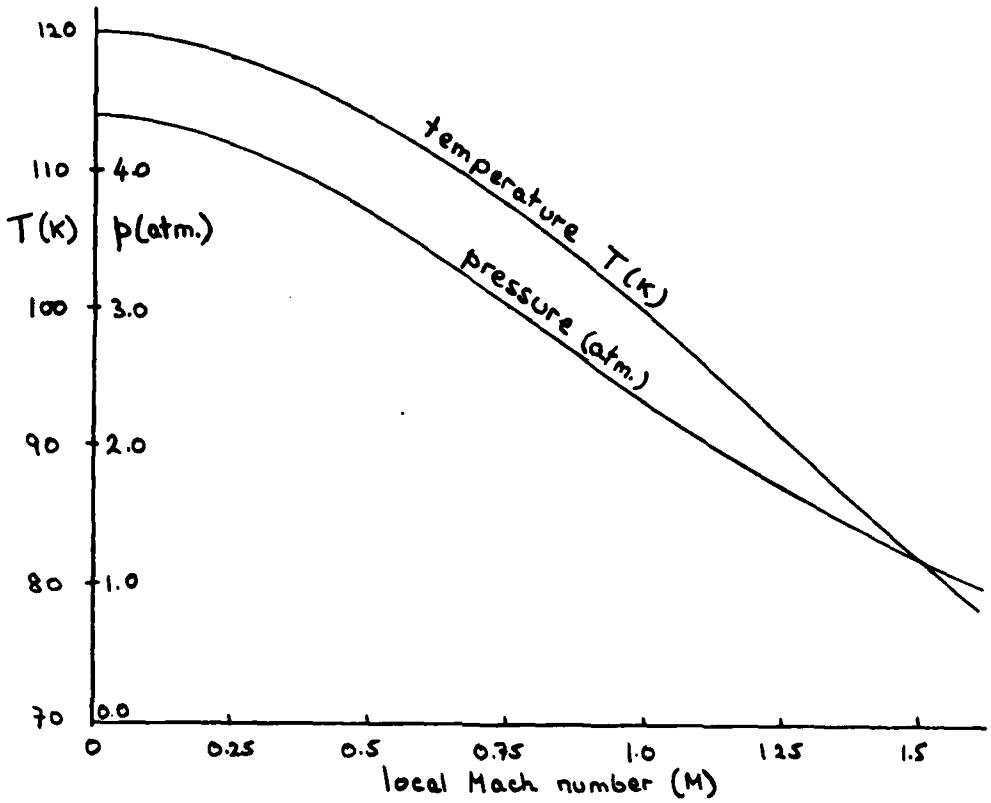

Fig 4 Homentropic expansion of nitrogen

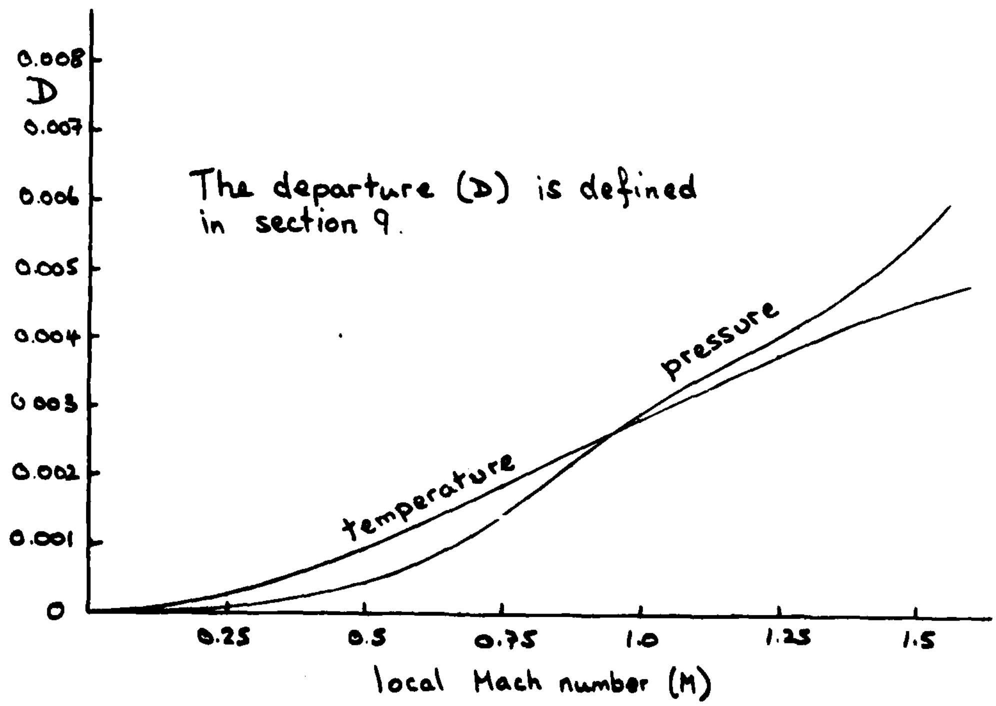

Fig 5 Departure from ideal gas

REPORT DOCUMENTATION PAGE

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17. Abstract

As a contribution to the investigation of the suitability of using cryogenic nitrogen as the test gas in a high Reynolds number transonic wind-tunnel, a study is made here of the real gas effects of nitrogen at low temperatures. The study, which is limited to inviscid, homentropic flow of a non-conducting gas, takes the form of an independent confirmation of results by Kilgore *et al*[1]. A recent paper by Wagner and Schmidt[2] on this subject employs a different equation of state from that used here and their investigations cover more than just homentropic flow. The new contribution in this Memorandum is that the use of a simplified equation of state enables an expression for enthalpy (and hence the terms in Bernoulli's equation) to be derived by analytic integration.

A shortened version of this Memorandum was presented at the First International Symposium on Cryogenic Wind-Tunnels at Southampton University 3-5 April 1979.

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